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# Field strength correlators in full QCD *

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## Abstract

We study, by numerical simulations on a lattice, the behaviour of the gauge–invariant two–point correlation functions of the gauge field strengths in the QCD vacuum with dynamical fermions.
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## 1. Introduction

A relevant role in hadron physics is played by the gauge–invariant two–point correlators of the field strengths in the QCD vacuum. They are defined as

$$\mathcal{D}_{\mu\rho,\nu\sigma}(x) = \langle 0|\mathrm{Tr}\left\{G_{\mu\rho}(x)S(x,0)G_{\nu\sigma}(0)S^{\dagger}(x,0)\right\}|0\rangle \ , \tag{1.1}$$

where $G_{\mu\rho} = gT^a G^a_{\mu\rho}$ is the field–strength tensor and $S(x,0)$ is the Schwinger phase operator needed to parallel–transport the tensor $G_{\nu\sigma}(0)$ to the point $x$.

They govern the effect of the gluon condensate on the level splittings in the spectrum of heavy $Q\bar{Q}$ bound states [1, 2, 3]. They are the basic quantities in models of stochastic confinement of colour [4, 5, 6] and in the description of high–energy hadron scattering [7, 8, 9, 10].

These correlators have been determined on the lattice in the *quenched* (i.e., pure–gauge) theory, with gauge group $SU(2)$ [11], and also in the *quenched* $SU(3)$ theory in the range of physical distances between 0.1 and 1 fm [12, 13]. In this paper we compute them in *full* QCD, i.e., we also include the effects of dynamical fermions.

The technique used is the same as in Refs. [12, 13]. The basic idea is to remove the effects of short–range fluctuations on large distance correlators by a local *cooling* procedure [14, 15]. Freezing the links of QCD configurations one after the other, damps very rapidly the modes of short wavelength, but requires a number $n$ of cooling steps proportional to the square of the distance $d$ in lattice units to affect modes of wavelength $d$:

$$n \simeq kd^2 \ . \tag{1.2}$$

Cooling is a kind of diffusion process. If $d$ is sufficiently large, there will be a range of values of $n$ in which lattice artefacts due to short–range fluctuations have been removed, without touching the physics at distance $d$. This removal will show up as a plateau in the dependence of the correlators on $n$.

The results are presented in Sect. 2. The determination was done at $\beta = 5.35$ ($\beta = 6/g^2$, where $g$ is the coupling constant) on a $16^3 \times 24$ lattice with four flavours of *staggered* fermions and the Wilson action for the pure–gauge sector. We have used a standard hybrid Monte Carlo (HMC) algorithm, in particular the so–called $\Phi$ algorithm described in detail

in Ref. [16]: the trajectory length $\tau$ was taken to be 0.3 with a molecular–dynamics step size $\delta\tau = 0.004$. The bare quark mass was chosen to be $a \cdot m_q = 0.01$ ($a$ being the lattice spacing), which should be a reasonable approximation to the chiral limit. A determination was also made for $a \cdot m_q = 0.02$, which we shall comment in the following. In Sect. 3 we discuss our results and give some concluding remarks.

## 2. Computations and results

The parametrization of the correlators is taken from Refs. [4, 5, 6]:

$$\begin{aligned} \mathcal{D}_{\mu\rho,\nu\sigma}(x) = {} & (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}) \left[ \mathcal{D}(x^2) + \mathcal{D}_1(x^2) \right] \\ & + (x_\mu x_\nu \delta_{\rho\sigma} - x_\mu x_\sigma \delta_{\rho\nu} + x_\rho x_\sigma \delta_{\mu\nu} - x_\rho x_\nu \delta_{\mu\sigma}) \frac{\partial \mathcal{D}_1(x^2)}{\partial x^2} \ . \end{aligned} \tag{2.1}$$

$\mathcal{D}$ and $\mathcal{D}_1$ are invariant functions of $x^2$. We work in the Euclidean theory.

It is convenient to define a $\mathcal{D}_\parallel(x^2)$ and a $\mathcal{D}_\perp(x^2)$ as follows:

$$\begin{aligned} \mathcal{D}_\parallel & \equiv \mathcal{D} + \mathcal{D}_1 + x^2 \frac{\partial \mathcal{D}_1}{\partial x^2} \ , \\ \mathcal{D}_\perp & \equiv \mathcal{D} + \mathcal{D}_1 \ . \end{aligned} \tag{2.2}$$

On the lattice we can define a lattice operator $\mathcal{D}^L_{\mu\rho,\nu\sigma}$, which is proportional to $\mathcal{D}_{\mu\rho,\nu\sigma}$ in the naïve continuum limit, i.e., when the lattice spacing $a \to 0$ [12, 13]. Making use of the definition (2.2) we can thus write, in the same limit,

$$\begin{aligned} \mathcal{D}^L_\parallel(\hat{d}a) & \underset{a\to 0}{\sim} a^4 \mathcal{D}_\parallel(d^2a^2) + \mathcal{O}(a^6) \ , \\ \mathcal{D}^L_\perp(\hat{d}a) & \underset{a\to 0}{\sim} a^4 \mathcal{D}_\perp(d^2a^2) + \mathcal{O}(a^6) \ . \end{aligned} \tag{2.3}$$

Higher orders in $a$ in Eq. (2.3) as well as possible multiplicative renormalizations are removed by cooling the quantum fluctuations at the scale of the lattice spacing, as explained in the Introduction.

The only scale in our system is the lattice spacing $a$: its value in physical units depends on $\beta$. Here we work with only one value of $\beta$, so we could present our results directly in

units of $a$. However, in order to facilitate the comparison with our previous works [12, 13] we shall use the familiar parametrization

$$a(\beta) = \frac{1}{\Lambda_F} f(\beta) \; , \tag{2.4}$$

where the *scaling function* $f(\beta)$ is given by the usual two–loop expression:

$$f(\beta) = \left( \frac{8}{25} \, \pi^2 \beta \right)^{231/625} \exp \left( - \frac{4}{25} \pi^2 \beta \right) \; , \tag{2.5}$$

for gauge group $SU(3)$ and $N_f = 4$ flavours of quarks. $\Lambda_F$ in Eq. (2.4) is an effective $\Lambda$–parameter for QCD in the lattice renormalization scheme, with $N_f = 4$ flavours of quarks. With this parametrization:

$$\begin{aligned} \mathcal{D}^L_\parallel f(\beta)^{-4} &= \frac{1}{\Lambda_F^4} \mathcal{D}_\parallel \left( \frac{d^2}{\Lambda_F^2} f^2(\beta) \right) \; , \\ \mathcal{D}^L_\perp f(\beta)^{-4} &= \frac{1}{\Lambda_F^4} \mathcal{D}_\perp \left( \frac{d^2}{\Lambda_F^2} f^2(\beta) \right) \; . \end{aligned} \tag{2.6}$$

We have measured the correlations on a $16^3 \times 24$ lattice at distances $d$ ranging from 3 to 8 lattice spacings and at $\beta = 5.35$. At this value of $\beta$ the lattice spacing $a(\beta)$, extracted from the string tension or the $\rho$ mass, is of the order of 0.11 fm [17, 18], so that the lattice size is approximately 2 fm and therefore safe from infrared artefacts. In Fig. 1 we display the results for $\mathcal{D}^L_\parallel f(\beta)^{-4}$ and $\mathcal{D}^L_\perp f(\beta)^{-4}$ versus $d_{\rm phys} = (d/\Lambda_F) \; f(\beta)$, for a quark mass $a \cdot m_q = 0.01$. Measurements have been done on a sample of 150 configurations, each separated by 15 HMC trajectories. Statistical errors have been estimated by using a standard blocking procedure. As in Ref. [13] we have tried a best fit to these data with the functions

$$\begin{aligned} \mathcal{D}(x^2) &= A_0 \exp\left(-|x|/\lambda_A\right) + \frac{a_0}{|x|^4} \exp\left(-|x|/\lambda_a\right) \; , \\ \mathcal{D}_1(x^2) &= A_1 \exp\left(-|x|/\lambda_A\right) + \frac{a_1}{|x|^4} \exp\left(-|x|/\lambda_a\right) \; . \end{aligned} \tag{2.7}$$

We have obtained the following results:

$$\begin{aligned} \frac{A_0}{\Lambda_F^4} = (1.74 \pm 0.24) \times 10^{10} \quad &, \quad \frac{A_1}{\Lambda_F^4} = (0.20 \pm 0.10) \times 10^{10} \; , \\ a_0 = 0.71 \pm 0.03 \quad &, \quad a_1 = 0.45 \pm 0.03 \; , \\ \frac{1}{\lambda_A \Lambda_F} = 544 \pm 27 \quad &, \quad \frac{1}{\lambda_a \Lambda_F} = 42 \pm 11 \; , \end{aligned} \tag{2.8}$$

with $\chi^2/N_{\rm d.o.f.} \simeq 0.5$. The continuum lines in Fig. 1 correspond to the central values of this best fit.

The corresponding results for the quark mass $a \cdot m_q = 0.02$ are displayed in Fig. 2, using for $\Lambda_F$ the same value adopted for $a \cdot m_q = 0.01$: we shall comment on this point in the next Section. Measurements have been done on a sample of 30 configurations, each separated by 20 HMC trajectories. A best fit to these data with the same functions (2.7) gives the results

$$\begin{aligned}
\frac{A_0}{\Lambda_F^4} &= (3.48 \pm 0.42) \times 10^{10} \quad , \quad \frac{A_1}{\Lambda_F^4} = (0.46 \pm 0.21) \times 10^{10} \ , \\
a_0 &= 0.66 \pm 0.03 \quad , \quad a_1 = 0.39 \pm 0.03 \ , \\
\frac{1}{\lambda_A \Lambda_F} &= 631 \pm 23 \quad , \quad \frac{1}{\lambda_a \Lambda_F} = 61 \pm 20 \ ,
\end{aligned} \tag{2.9}$$

with $\chi^2/N_{\rm d.o.f.} \simeq 0.7$. Again, the continuum lines in Fig. 2 correspond to the central values of this best fit.

## 3. Discussion

Two quantities of physical interest can be extracted from our lattice determinations:

1) the correlation length $\lambda_A$ of the gluon field strengths, defined in Eq. (2.7);

2) the so–called *gluon condensate*, defined as

$$G_2 \equiv \langle \frac{\alpha_s}{\pi} : G^a_{\mu\nu} G^a_{\mu\nu} : \rangle \qquad (\alpha_s = \frac{g^2}{4\pi}) \ . \tag{3.1}$$

Both of them play an important role in phenomenology. The correlation length $\lambda_A$ is relevant for the description of vacuum models [4, 5, 6]. The relevance of the gluon condensate was first pointed out by Shifman, Vainshtein and Zakharov (SVZ) [19]. It is a fundamental quantity in QCD, in the context of the SVZ sum rules.

From lattice we extract $\lambda_A$ in units of lattice spacing $a$. To convert these units to physical units, the scale must be set by comparison with some physical quantity. This

was done in Refs. [17, 18] by computing the string tension and the $\rho$ mass on the lattice and comparing them with the physical values. Their lattice was identical to ours ($16^3 \times 24$): they also used the same value of $\beta$ (5.35), as well as $N_f = 4$ flavours of staggered fermions with mass $a \cdot m_q = 0.01$, as we did. Their estimate for the lattice spacing is $a \simeq 0.11 \pm 0.01$ fm. This gives:

$$\lambda_A = 0.34 \pm 0.02 \pm 0.03 \, \mathrm{fm} \qquad (a \cdot m_q = 0.01) \ . \tag{3.2}$$

The first error comes from our determination [Eq. (2.8)], the second from the error in converting the lattice spacing to physical units. From $a \cdot m_q = 0.01$ to $a \cdot m_q = 0.02$ the value of the effective $\Lambda_F$, defined by Eqs. (2.4) and (2.5), can change in principle. Anyway, no published determination of $\Lambda_F$ (i.e., of the lattice spacing in physical units) exists for $a \cdot m_q = 0.02$. Some data on the pseudoscalar and vector meson masses, for quark masses $a \cdot m_q$ larger than 0.01, have been published in Ref. [20]. We have tried to extract $\Lambda_F$ (i.e., the lattice spacing) from those data by using the same procedure of Ref. [17]. We estimate that from $a \cdot m_q = 0.01$ to $a \cdot m_q = 0.02$ the effective mass–scale $\Lambda_F$ does not change appreciably within the errors. Assuming, as an indication, the same value of $\Lambda_F$ as for $a \cdot m_q = 0.01$, we then get:

$$\lambda_A = 0.29 \pm 0.01 \pm 0.03 \, \mathrm{fm} \qquad (a \cdot m_q = 0.02) \ . \tag{3.3}$$

The values (3.2) and (3.3) must be compared with the *quenched* value [12, 13]

$$\lambda_A = 0.22 \pm 0.01 \pm 0.02 \, \mathrm{fm} \qquad (\mathrm{YM\ theory}) \ . \tag{3.4}$$

Here the value $\Lambda_L \simeq 4.9 \pm 0.5$ MeV has been assumed for the pure–gauge $\Lambda$–parameter [21]. The correlation length $\lambda_A$ increases when going from chiral to *quenched* QCD and this tendency is confirmed by the fact that $\lambda_A$ decreases by increasing the quark mass. Of course, a precise determination of $\lambda_A$ should be done with more realistic values for the quark masses.

We now come to the gluon condensate. Our lattice provides us with a regularized determination of the correlators. At small distances $x$ a Wilson *operator–product–expansion* (OPE) [22] is expected to hold. The regularized correlators will then mix to the identity operator $\mathbf{1}$, to the renormalized local operators of dimension four, $\frac{\alpha_s}{\pi} {:}G^a_{\mu\nu} G^a_{\mu\nu}{:}$ and $m_f {:}\bar{q}_f q_f{:}$ ($f = 1, \ldots, N_f$, $N_f$ being the number of quark flavours), and to operators of

higher dimension:

$$\frac{1}{2\pi^2}\mathcal{D}_{\mu\nu,\mu\nu}(x) \underset{x\to 0}{\sim} C_{\mathbf{1}}(x)\langle\mathbf{1}\rangle + C_g(x)G_2 + \sum_{f=1}^{N_f} C_f(x) m_f \langle : \bar{q}_f q_f : \rangle + \ldots \ . \tag{3.5}$$

The mixing to the identity operator $C_{\mathbf{1}}(x)$ shows up as a $c/|x|^4$ behaviour at small $x$. The mixings to the operators of dimension four $C_g(x)$ and $C_f(x)$ are expected to behave as constants for $x \to 0$, while the other Wilson coefficients in the OPE (3.5) are expected to vanish when $x \to 0$ (for dimensional reasons). The coefficients of the Wilson expansion are usually determined in perturbation theory and are known to be plagued by the so–called *infrared renormalons* (see for example Ref. [23] and references therein). In our case this means that, due to the infrared renormalon pole, terms coming from the mixing to the identity operator can produce by resummation a term which simulates a mixing to a condensate [23]. There is no specific recipe to disentangle this "*perturbative*" contribution to the condensates from a possible "*genuine*" value of them. A similar problem is present in any Wilson OPE in QCD, and in particular in the expansion which leads to the SVZ sum rules [19].

A practical way out, which provides good results for the sum rules, is to assume that the leading perturbative determination of the Wilson coefficients is a good approximation to the unknown determination which should be done by perturbing around the real vacuum of the theory (see for example Ref. [24] for a discussion about this point). In this spirit, we shall assume that the renormalon ambiguity can be safely neglected in the extrapolation for $x \to 0$ of our correlators. With the normalization of Eq. (3.5), this gives $C_g(0) \simeq 1$. On the same line, the contribution from the quark operators in (3.5) can be neglected, because the corresponding condensates $m_f\langle : \bar{q}_f q_f : \rangle$ are much smaller than $G_2$ and the mixing coefficients $C_f(x)$ are of higher order than $C_g(x)$ in the perturbative expansion. Within these approximations, we get the following expression for the gluon condensate, in terms of the parameters defined in Eq. (2.7):

$$G_2 \simeq \frac{6}{\pi^2}(A_0 + A_1) \ . \tag{3.6}$$

At $a \cdot m_q = 0.01$ this gives, in physical units,

$$G_2 = 0.015 \pm 0.003\,^{+0.006}_{-0.003}\,\mathrm{GeV}^4 \qquad (a \cdot m_q = 0.01) \ . \tag{3.7}$$

At $a \cdot m_q = 0.02$ we obtain:

$$G_2 = 0.031 \pm 0.005\,^{+0.012}_{-0.007}\,\mathrm{GeV}^4 \qquad (a \cdot m_q = 0.02)\ . \tag{3.8}$$

The two values (3.7) and (3.8) should be compared with the corresponding value in the *quenched* theory [13]:

$$G_2 = 0.14 \pm 0.02\,^{+0.06}_{-0.05}\,\mathrm{GeV}^4 \qquad (\mathrm{YM\ theory})\ . \tag{3.9}$$

The gluon condensate $G_2$ appears to increase with the quark mass, as expected, tending towards the asymptotic (pure–gauge) value of Eq. (3.9). Contrary to the previous discussion for $\lambda_A$, we have here a theoretical tool to understand the dependence of $G_2$ on the quark masses. According to Ref. [25], we expect the following low–energy theorem to hold for small quark masses ($m_f \ll \mu$, $\mu$ being the renormalization scale):

$$\frac{d}{dm_f}\langle\frac{\alpha_s}{\pi} : G^a_{\mu\nu} G^a_{\mu\nu} :\rangle = -\frac{24}{b}\langle : \bar{q}_f q_f :\rangle\ , \tag{3.10}$$

where $b = 11 - \frac{2}{3}N_f$, for a gauge group $SU(3)$ and $N_f$ quark flavours. For the two gluon condensates (3.7) and (3.8) one must use the renormalized quark masses $m_f$ [26] corresponding to $a \cdot m_q = 0.01$ and $a \cdot m_q = 0.02$ respectively: for $a \cdot m_q = 0.01$ we have approximately $m_f \simeq 44$ MeV. Making use of the popular values for the quark condensate ($\langle\bar{q}q\rangle \simeq -0.013\,\mathrm{GeV}^4$ [17, 27]) and for the physical quark masses ($m_u \simeq 4$ MeV, $m_d \simeq 7$ MeV and $m_s \simeq 150$ MeV), we can thus extrapolate from the value (3.7) to the *physical* gluon condensate, obtaining:

$$G_2^{(physical)} \sim 0.022\,\mathrm{GeV}^4\ . \tag{3.11}$$

The procedure used is the same as in Ref. [25]. The prediction (3.11) agrees with the empiric value obtained from experiments [27, 28]: $G_2^{(empiric)} \simeq 0.024 \pm 0.011\,\mathrm{GeV}^4$.

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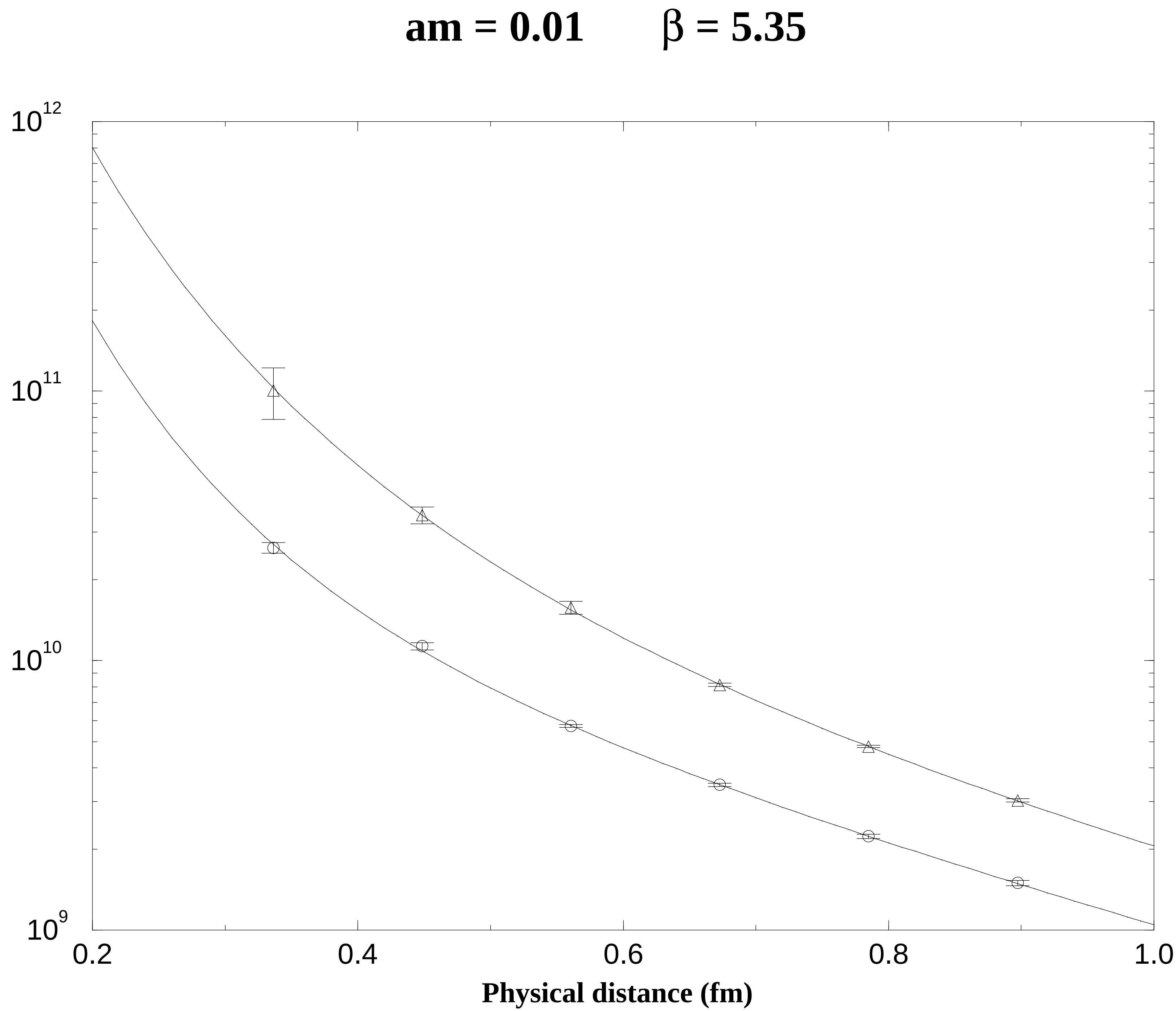
## FIGURE CAPTIONS

**Fig. 1.** The functions $\mathcal{D}_{\perp}^{L} f(\beta)^{-4}$ (upper curve) and $\mathcal{D}_{\parallel}^{L} f(\beta)^{-4}$ (lower curve) versus physical distance, for quark mass $a \cdot m_q = 0.01$. The curves correspond to our best fit [Eqs. (2.7) and (2.8)].

**Fig. 2.** The same as in Fig. 1 for quark mass $a \cdot m_q = 0.02$. The curves correspond to our best fit [Eqs. (2.7) and (2.9)].


am = 0.01 β = 5.35
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0.2
0.4
0.6
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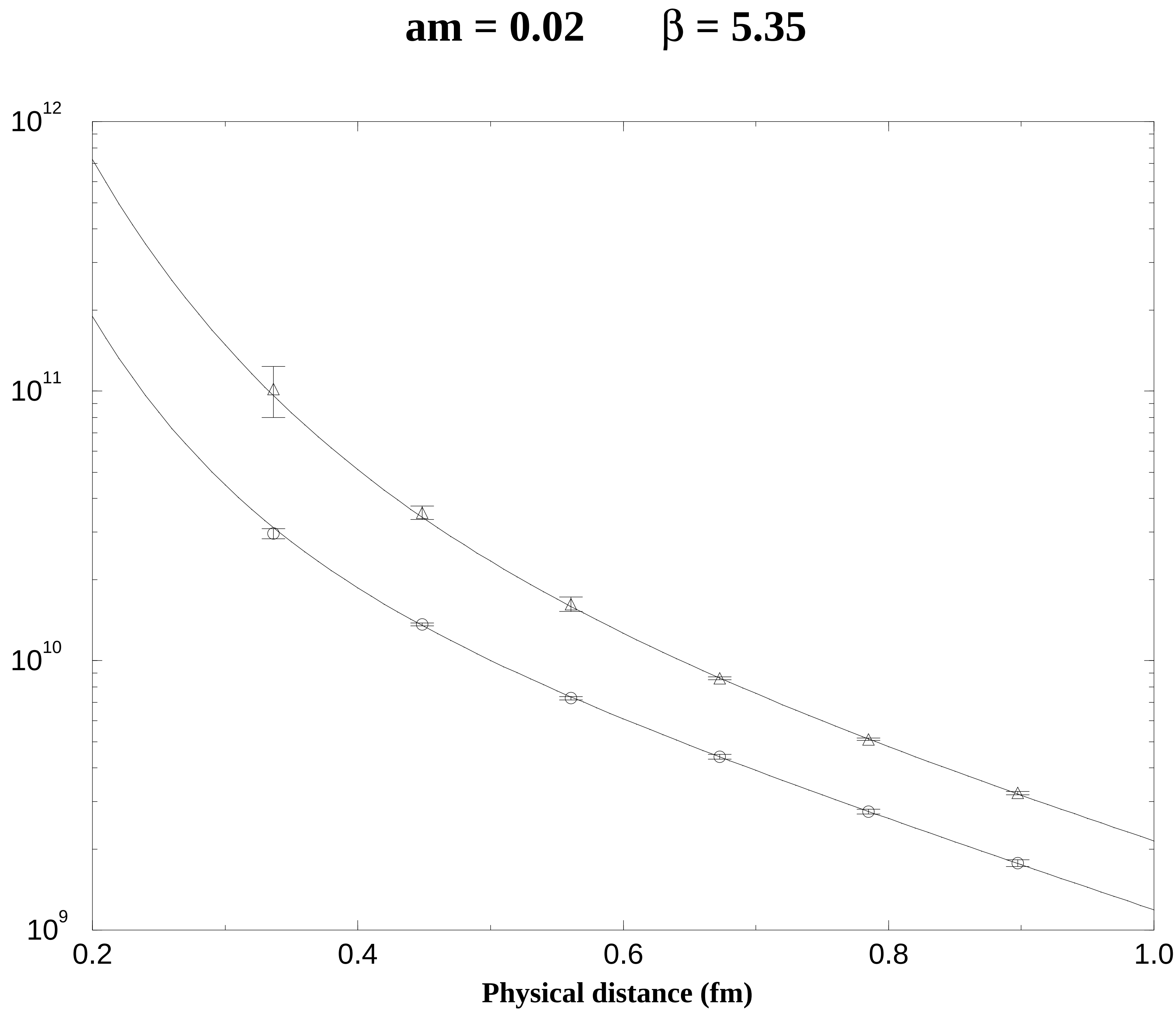
1.0
Physical distance (fm)



am = 0.02 β = 5.35
10^12
10^11
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Physical distance (fm)